THE

SCIENTIFIC JOURNAL.

NEW-YORK, APRIL, 1818.

Method of preventing, or curing, the damp in houses.

Mr. Editor,

The best use of philosophy consists in the application of scientific discoveries, to improve and ameliorate the condition of mankind; this is my sole reason for sending you the following remarks, on a subject which, I have no doubt, you will agree with me in thinking of considerable importance, at least, to a considerable portion of society.

Not far from my place of residence, two females died in the same week, in consequence of having lived in damp houses; and the same causes must produce similar effects, in every part of this extensive country.

Damp houses, by suddenly checking perspiration, contribute greatly to the production of hectic and other fevers, which frequently terminate in death. Persons of strong constitutions, who, after being heated, sit to cool, or sleep in such houses, may date the origin of such diseases, as above mentioned, to such circumstances; what then must be the effect produced on the delicate female in such a situation? It will sometimes happen, that dwelling houses must be erected, where it is physically impossible for every part to be dry, or free from damp, unless some proper methods be adopted to secure that desirable

object. The best method which I have seen, is, to remove part of the back ground, when the house is built on the slope of a hill of slight elevation, to a level with the foundation. But this method, which is often expensive, is frequently insufficient effectually to remove the cause ; for the stagnant air, in the area so sunk, holds much water in solution, which will of course saturate the outer walls and adjoining rooms with moisture.

The method of prevention, or cure, which I beg leave to propose, is as follows: In cases, where the damp proceeds from one, or more walls, or sides of the houses, being built on low ground, the side of a hill, &c. let a thin inner wall be built air-tight, parallel to, and at a small distance from the outer wall, and continued above and below the extent of the damp. Air-holes must be left through the outer wall into the top of the inclosed space between the walls ; and from the bottom of such space, in the middle of its breadth, a metallic pipe. or any kind of flue, is to be carried under any convenient part of the ground floor, to terminate in the kitchen, or principal fire-place. The air in the mouth of the flue, or tube, thus becomes rarified by the fire, and a constant stream will issue from the atmosphere, through the outer apertures and the flue, which will completely carry off the damp, and, increasing the draft of the chimney, will prevent it from smoking. If the damp proceeds from the ground, and is occasioned by the house being built on a humid flat, it will be necessary to make the ground floor double, with an apparatus, as above described. And, where there are drains of running water, a current of air must, by the same means, be carried over them to the fire, observing to make the current of air a little wider than that of the water below.

The above is the best method of preventing the dry rot, in buildings, which has ever yet been discovered.

Antiquity of Clock Making.

THE first clock with wheels which was known in France, was that given to Pepin the Short, by Paul I. In 807, the friend and protector of the arts in the east, Haraun al Raschid, presented to Charlemagne a clock, of which the historians of the times speak with great admiration; these clocks were imitated by the Italians. To Gerbert d'Aurillac, preceptor to Otho III, is attributed the invention of a clock, the movement of which was regulated by a balance. The clock of the palais was the first which Paris possessed: it was made by Henry de Vic, who was sent for by Charles V, from Germany; that of the church of Lyons, by Nicholas Lippius; and that of Strasburg, and of Lund in Sweden, so much praised by Derham, show the rapid progress which the art had made in the sixteenth and seventeenth centuries, and lead us to that perfection to which it arrived about the middle of the last century. The English invented the watch, and repeating pendulums. *Peuchet's Dictionnaire Universel, &c.*

CURIOSUS.

To the Editor of the Scientific Journal.

THE following description of a subterranean spring, which discharges itself into the Larian lake, and the subsequent reasoning on it, are contained in a letter from Pliny to Licinius, and by your permission are submitted to the perusal of the readers of the Journal, in hopes that some of your numerous readers will favour us with their opinions, as to which is the most favourable of Pliny's suppositions, or, what are the causes which produce the regular and surprising effects related by him.

Yours, &c. P. THOMPSON.

Pliny to Licinius.

"I have brought you, as a present out of the country, a query, which well deserves the consideration of your extensive erudition. There is a spring, which rises in a neighbouring mountain, and running among the rocks, is received into a little banquetting-room, whence, after being detained a short time, it falls into the Larian lake. The nature of this spring is extremely surprising; it ebbs and flows regularly *three* times a day. This increase, and decrease, are plainly visible, and very entertaining to observe. You sit down by the side of the fountain, and while you are taking a repast, and drinking its water, which is extremely cool, you see it gradually rise and fall. If you place a ring, or any thing else, at the bottom when it is dry, the stream reaches it by degrees, till it is entirely covered, and then again gently retires from it; and this you may see it do for three times successively. Shall we say, that some secret current of air stops and opens the fountain head as it advances to, and recedes from it, as we see in bottles, or other vessels of that nature, where there is not a free and open passage, though you turn their necks downwards, yet, the outward air obstructing the vent, they discharge their contents, as it were, by starts? Or, may it not be accounted for upon the same principle as the flux and reflux of the sea? Or, as those rivers which discharge themselves into the sea, meeting with contrary winds, and the swell of the ocean, are forced back in their channels; so, may there not be something that checks this fountain, for a time, in its progress? Or, is there rather a certain reservoir, that contains these waters in the bowels of the earth, which, while it is recruiting its discharges, the stream flows more slowly, and in less quantity, but, when it has collected its due measure, it runs again in its usual strength and fulness? Or, lastly, is there not, I know not what kind of subterranean poise that throws up the water when the fountain is dry, and repels it

when it is full? You, who are so well qualified for the inquiry, will examine the reasons for this wonderful appearance; it will be sufficient for me, if I have given you a clear description. Farewell."

CHEMICAL EXPERIMENTS,

FOR THE USE OF STUDENTS.

Ex. 1. *Two liquors, which are cold, will, when mixed together, become hot.*

Thus, into a phial containing a little cold water, pour some sulphuric acid, and a considerable sensation of heat will be perceived. By pouring in more acid, a greater degree of heat may be generated; and the mixture may be raised to a degree of heat much greater than that of boiling water.

Ex. 2. *To produce combustion with two solid bodies and a fluid.*

Upon a mixture of equal parts of sugar and oxymuriate of potash, let fall a drop of sulphuric acid, and combustion will immediately take place.

Ex. 3. *To produce combustion under water.*

Take a bit of phosphorus, about the size of a small pea, and lay it upon a few grains of oxymuriate of potash, at the bottom of a glass, poor some water gently into the glass, so as not to mix them. Put some sulphuric acid into the bowl of a tobacco pipe, and immerse the small end of the pipe into the water, nearly to the bottom of the glass, and stir it gently round, the water will appear to be on fire. This experiment should be made, *with some caution*, in a dark room.

Ex. 4. Put a piece of phosphorus, about the size of a small marble, into a Florence flask nearly half full of water; place the flask over a spirit lamp, and make the water boil: beautiful corruscations will arise, of a bluish colour, and balls of fire will be thrown from the water, in the manner of artificial fire-works.

Ex. 5. *To change and restore colours.*

Dilute syrup of roses with a little pure water, to which add a few drops of a solution of potash, and it will become green; to this green liquid, add a few drops of sulphuric acid, and the red colour will be again restored.

Ex. 6. *To produce combustion and inflammation.*

Take some very dry charcoal, reduce it to a very fine powder, and put it into a warm tea-cup; pour upon it some strong nitrous acid, and combustion and inflammation will ensue.

Ex. 7. Put some fresh calcined magnesia into a tea-cup, and place it on the hearth; pour over it as much concentrated sulphuric acid as will cover the magnesia, sparks will instantly be thrown out, and the mixture will be completely ignited.

Ex. 8. Dissolve a little phosphorus in alcohol, pour a little of the mixture into a glass of warm water, a flame instantly makes its appearance, and waves beautifully on the surface of the water.

Ex. 9. *To make phosphorus bottles.*

Put a piece of phosphorus, about the size of a pea, into a small phial, fill it about half full with olive oil, or oil of almonds, and cork it up. When the cork is taken out in the night, sufficient light will be evolved, to show the hour by a watch. These bottles should not be played with, because, if

the phosphorus get upon a person's hand, it will burn him desperately.

Ex. 10. *To make a metallic tree.*

Dissolve an ounce of acetate of lead, in about a quart of water, and filter the solution. Put this into a bottle, and suspend a piece of zinc in it, by means of a brass wire ; a decomposition of the salt will commence, the lead will be set at liberty, will attach itself to the zinc, and form a metallic tree.

Ex. 11. *Liquid combustion.*

Pour some phosphuretted ether upon a lump of sugar, and drop it into a glass of warm water ; the surface of the water will soon become luminous, and brilliant undulations will be produced, exhibiting the appearance of liquid combustion.

Ex. 12. Into a glass tumbler of pure water, put a small piece or two of phosphuret of lime ; in a short time *flashes of fire* will dart from the surface of the water, and terminate in ringlets of smoke, ascending in regular succession.

Some of these experiments have been selected from Park's Catechism, and others from Thompson's Chemistry ; they will be found useful to such young persons as are fond of the study of chemistry, especially as they may be made with a very simple apparatus. We beg leave to observe to students, that though experiments are necessary in chemical researches, yet, experiments alone are not sufficient ; they must endeavour to discover the reason of the different phenomena produced, by carefully studying the theory.

Those gentlemen who are in the practice of making chemical or philosophical experiments, will oblige the editor, by transmitting an account of their discoveries to the Scientific Journal.

TESTS FOR DIFFERENT METALS.

Gold. If an infusion of nutgalls be added to a solution of gold, a green colour is formed, and in a short time afterwards, a brown precipitate appears at the bottom of the liquor.

A plate of tin, immersed into a solution containing gold, occasions a purple coloured powder to be precipitated.

Sulphate of iron precipitates gold, in the metallic state.

Silver is easily discovered, when contained in solution by muriat of soda, throwing down a copious white precipitate, which is the muriat of silver.

A solution of the sulphat of iron precipitates silver, in the metallic state.

Mercury is usually detected by pouring into the solution an infusion of nutgalls; an orange-yellow precipitate immediately appears.

Oxy-muriat of mercury, (corrosive sublimate) is known to be present, when potash, or soda, occasion a brick-red precipitate.

A plate of copper, immersed into a liquid mercurial salt, precipitates running mercury.

Copper. When only a small portion of copper is contained in solution, liquor of ammonia (hartshorn water) occasions that solution to assume a beautiful blue colour.

A brown precipitate is produced, when Gallic acid is poured into a liquid containing a cupreous salt.

When a piece of iron is plunged into a liquid holding any cupreous salt, the copper is precipitated in the metallic state.

Iron. Prussiate of lime, when mixed with a solution containing iron, produces a copious blue precipitate.

When a small quantity of the infusion of nutgalls is poured into a liquid holding iron, a black sediment is immediately generated.

Tin is readily distinguished, when in solution, by the following process: pour a solution of the corrosive sublimate into the liquor suspected, when, if it contain the protoxide, a black precipitate is directly thrown down; but if the peroxide be present, a white one will be produced.

Lead. The infusion of nutgalls occasions a white precipitate, from solutions containing lead; sulphate of soda, also, throws down a copious white precipitate.

A piece of zinc being suspended in a solution of lead, makes either the lead in its metallic state, or a white precipitate, to appear.

Antimony. A plate of iron, or zinc, immersed into antimonial solutions, occasions a black powder to be precipitated in great abundance.

Antimony is generally separated from its solutions in a white precipitate, by the infusion of nutgalls.

PHILOSOPHICAL QUESTIONS, ANSWERED.

Qu. 1.

Mr. Hirst says, "If the edge of a piece of pure silk be exposed to the flame of a candle, it burns to a black coal, after the manner of wool, or hair; but if there be any cotton, or linen, mixed with it, they may be discovered, for they will not burn to a coal, but are turned to ashes in a moment, accompanied with a different kind of smoke, and by this means both the warp and woof may be examined, by taking a thread of each."

Mr. Harrison observes, that, " Cotton is extremely combustible, and burns with a clear, lively flame. Silk, on the contrary, is but imperfectly combustible, though fire rapidly blackens and discomposes it; these distinguishing characteristics are a sufficient answer to the question."

Qu. 2.

Mr. J. Nowell says, " The electric fluid, like caloric, possesses that most curious property, of penetrating the most compact bodies, and of passing through them with greater facility than through those bodies which, on account of their porosity, one would imagine would admit them more easily. From the similarity of the two fluids, in this respect, and from their uniform similarity in many others, one might be led to suppose, that their radical properties are of the same nature. If, however, we suppose them to be radically similar, various contrary properties suggest them to be only modifications of each other. That property of the electric fluid, of being conducted by the most dense bodies, agrees equally with the properties of caloric in this respect, and it is very probable, reasoning from analogy, that each are conducted through such bodies on one and the same principle. Taking this, then, for granted, the same theory which is given to account for the difference that bodies possess for conducting caloric, will, it is evident, equally apply to the facts under consideration. Bodies, whether in a solid, liquid, or æriform state, are composed of particles, or atoms, not indeed cohering together, but kept asunder by an unknown cause, called repulsion. Now, the greater the action of this repulsive power, the greater must be the distance of the particles from each other; and this law evidently constitutes the difference between dense and light bodies. If the particles of bodies, then, be not in a state of actual cohesion, as might easily be proved, between these particles there must exist a space, either void, or filled up with some subtle fluid, such as atmospheric air; conse-

quently, if either caloric, or the electric fluid, was given by any means to a light body, it would be imperfectly conducted by that body, because the void spaces left by the atoms, not cohering closely, would act as imperfect, if not as *non* conductors, and by this means would considerably impede the progress of the electric fluid, or the caloric. Now, with iron it would be the reverse, for the repulsive power of the particles being weak, and, of course, not containing so much of the void, or non-conducting medium, little resistance would be opposed to the particles of either the electric fluid, or caloric, in their passage through it. From the facts here detailed, we readily perceive that the conducting powers of bodies increase exactly in the same ratio as the repulsive power of their particles decrease; so that if, by any means, we could solidify bodies, by diminishing the repulsive power of their particles, their capacity for conducting the electric fluid would be increased. Hence the statement made in the question is manifest.

Qu. 3.

J. H. N. observes, "The sweet fluid called honey-dew does not fall from the air, as some conjecture, but is excerned by insects of the *aphis genus*, which adhere to the under sides of the leaves, and emit this liquor upon the surfaces of other leaves below them. The insect exists by sucking the juices from the vessels which are in the leaves, and it is remarkable, that it rejects the saccharine part, or honey-dew, which drops from its body, in small, but sometimes perceptible globules, and so affords nourishment to myriads of ants, and other small insects. What a striking instance of the wisdom, power, and goodness, of God in the creation!"

Mr. Nowell says, "It is natural to suppose, that the substance called honey-dew was originally part of the sap of the plant. From analogy, we can hardly doubt, that nature has

provided proper vessels in plants, as well as in animals, for carrying off superabundant quantities of the circulating fluids, as occasion requires. Reasoning in this manner, we see how it is, that, at particular seasons of the year, this substance appears; for a super-quantity of sap may exist, which being given out by the vessels appropriated for that purpose, on the surface of the leaves, where, absorbing one of the component parts of the atmosphere, probably the oxygen, it becomes a new substance, namely, sugar, and perhaps mucilage; so exceedingly prone are vegetable substances to decomposition."

Mr. M. Harrison. "I have never observed what is called honey-dew on the dead, but only on the living leaves of trees; whereas, if it fell from the sky, as some suppose, it would light on all substances which came in its way, which does not happen. A vegetable is a collection of tubes, or vessels, endowed with life, able to imbibe particular fluids, and to alter their nature; in the same manner as animals, which secrete milk and fat from their food, so vegetables secrete gum, sugar, and various rosinous substances from their food, which is water and air, holding heterogeneous matter in solution; and these being absorbed by their roots, become sap, which is carried up into the leaves by vessels, called by Mr. Knight, central vessels, where it undergoes a chemical change, and becomes a new substance in its return down the bark. In the summer months, when the trees are full of foliage, the sap circulates much quicker than at other seasons of the year, and as trees are appropriated with vessels, by which they can discharge any surplus of sap with which they may be overcharged, what exudes, and remains upon the leaves of plants and trees, becomes sweet and glutinous; and when, by the action of the sun and air, the watery particles are evaporated, it forms what is termed honey-dew."

Mr. J. Baines. "The Abbe Boissier de Sauvages, in a memoir

read before the society of sciences at Montpelier, has given a very clear and circumstantial account of the origin of honey-dew. He saw it in its primitive state, on the leaves of the Holm oak, before the drops had run together, in such a form as to leave no doubt but that it had exuded from the leaves. Heat, he supposes, is the cause of its emission, for, though common heat only promotes the transpiration of the more volatile and fluid juices, yet, a sultry heat, especially if reflected by the clouds, may so far dilate the vessels as to produce a more viscous juice, such as the honey-dew. But there is another kind of honey-dew found on the leaves of plants and trees, which is ejected from the vine-fretter, (aphis,) and forms the most delicious honey known in nature."

Qu. 4.

Mr. R. T. says, "I suppose that brass in a lump will fuse in less time than the same quantity in filings, for this reason; caloric is able to pervade all bodies, and in proportion as bodies are good or bad conductors of electricity, they are good or bad conductors of caloric, and the more light or porous a body is, the worse conductor it is found to be. Now, brass filings occupy a greater space than the same weight in a lump; of course, the interstices of the filings are occupied by atmospheric air, which is a worse conductor of caloric than the metal; hence it may be inferred that, on this account, the metal is longer in becoming melted. Also, matter of every kind is regulated by attraction and repulsion, and fluidity is only an alteration in the situation of the particles, occasioned by a greater or less portion of caloric, which the body imbibes; now, in the filings, the caloric requires a longer time to overcome the cohesive attraction of the metal; and this, in my opinion, is the answer to the question."

Mr. Nowell refers to the theory contained in his solution to

the second question, above given. *Mr. J. Smith* observes, that the fusion of metals is accomplished by the action of caloric, which, by combining with solid substances, diminishes, or destroys the attractive power of the particles, and causes them to pass into a fluid state. Now, it is well known, that metal in a solid state is more permeable to caloric, or, in other words, is a better conductor of heat, than filings are; consequently, it will fuse with greater facility. *Mr. Savage* and *Mr. Hirst* give similar answers.

Qu. 5.

Mr. Thompson gives the following process for purifying fish oil. Take one gallon of crude stinking oil, and put to it a pint of water, poured off from two ounces of lime slacked in the air; stir the mixture several times for the first twenty-four hours, and when it has stood about a day more, the water will have sunk below the oil, which must be carefully separated from it.

A better Method. Take one gallon of crude stinking oil, and mix with it a quarter of an ounce of powdered chalk, a quarter of an ounce of lime slacked in the air, and half a pint of water; stir them together, and when they have stood some hours, add a pint of water, and two ounces of pearl ashes, and place the mixture over a fire that will keep it simmering, till the oil appears of a light amber colour, and has lost all smell, except a greasy soap-like scent. Then add half a pint of water, in which one ounce of salt has been dissolved, and having boiled it half an hour, pour the mixture into a proper vessel, and let it stand a few days, till the oil and water separate.

If this operation be repeated, diminishing the quantity of ingredients one half, the oil may be brought to a very light colour, and rendered equally sweet with the common spermaceti oil.

Oil purified in this manner, is found to burn much better, and

answers much better also for the purposes of the woollen manufacture. If oil be wanted thicker, and more unctuous, it may be rendered so by the addition of tallow, or fat.

Mr. Bamford says, "A patent was granted to a Mr. Collier, in 1798, for a chemical process for freeing fish oils from impurities of smell, taste, and colour, which was as follows: 'Pour a quantity of fish oil into a vessel heated to the temperature of 110 or 120 degrees of Fahrenheit's Thermometer, when a portion of caustic mineral alkali is to be added, the weight of which is to be four parts to the hundred of the oil. The mixture is then to be agitated, and, after the sediment and salt have subsided, it is to be poured off into another vessel, containing a quantity of finely pulverized fresh-burnt charcoal, and a small quantity of diluted sulphuric acid; the agitation must then be repeated, and when the charcoal, saline, and aqueous particles have subsided, the oil must be passed through strainers; by which it will be rendered perfectly transparent, and fit for use.'"

Qu. 6.

Mr. Nowell says, "Prussic acid holds soda, or potash, in a state of combination in a very feeble degree; even the weakest of the acids, the carbonic, is capable of decomposing these salts, and forming a carbonate with their respective bases. There is a triple salt of prussic acid, potash, and iron, which is much used as a test, and long since known by the name of Prussian alkali: this substance precipitates all the oxides of iron, in coloured precipitates, depending on the degree of oxidation, and is still capable of holding a limited portion of the same oxide in solution. A very curious fact, which completely baffled the sagacity of the eminent chemist, Mr. Proust. If from this trinary combination, or, as it is called, the triple prussiate of potash and iron, we could take the iron contained in a state of solution, its fixity and utility as a re-agent would be destroyed, and the car-

bonic acid of the atmosphere would, in a certain degree, decompose the salt, if the bottle was not well corked, drive off the acid, and combine with the base. Hence the proneness of the simple prussiate to decomposition. But if potash holds the prussic acid with a weak affinity, the case is not so with iron; it holds it with a strong affinity, and in fact is capable of taking it from its strongest combinations. Upon this datum we may form a theory, which will account for the fact mentioned in the question. We will suppose the triple prussiate made up of atoms arranged with particular symmetry; one atom of the salt, we will suppose is prussic acid, another potash, and a third iron; now, the attraction of the iron will be extended, when so minutely divided, even past the sphere of action of the potash; and if, by a combination of the base with carbonic acid, the prussic acid of the triple prussiate had a tendency to fly off, the superior attraction of the iron would hold it, and prevent its flight.

J. H. N., *Mr. J. Bamford*, and *Mr. R. T.* sent solutions.

To Question 7, we have not yet received any answer.

NEW PHILOSOPHICAL QUESTIONS.

Qu. 8. By *Miss M. Groves.*

What is the general opinion of philosophers, respecting the stony substances which have fallen to the earth at different times, and in various places?

Qu. 9. By *Mr. Thompson, Boston.*

Does dew arise, or descend?

Qu. 10. By *X.*

If water, or any other fluid, be strongly agitated, air bubbles will arise on the surface; these bubbles invariably as-

sume the form of the upper segment of a sphere; required the reason?

The two following questions are offered for solution to men of science; the time being unlimited. The first, we believe, has never been solved, and the second was proposed in the Stanford Mercury, in England, in A. D. 1817; and 50 pounds sterling was offered for a solution, in 6 months, but no solution appeared.

Qu. A. By *Themistocles.*

Writers on navigation assert, that the shortest distance between any two points on the surface of the earth, taken as a sphere, is the arc of a great circle; required the demonstration.

Qu. B. By *Mr. Joseph Waters, Graves Lane.*

Divide unity into three such positive parts, that if each part be increased by unity, the sums shall be three rational cubes.

MATHEMATICAL CORRESPONDENCE.

Solution to Qu. 4. By *Mr. William Wood, New-York.*

If, from the square of the first equation, the second be subtracted, and the remainder divided by 2, we shall obtain the sum of the rectangles

$$u\,x + u\,y + u\,z + x\,y + x\,z + y\,z = \left(\frac{a^2 - b}{2}\right) = p = 199.$$

Also, if from the cube of the first, we take the third, and divide by 3; and, again, from this subtract the product of the first and second, diminished by the third, we shall obtain,

$$2uxy + 2uyz + 2uxz + 2xyz = \frac{a^3 + 2c - 3ab}{3}.$$

This divided by 2, will give the sum of the solids, or

$$uxy + uyz + uxz + xyz = \left(\frac{a^3 + 2c - 3ab}{6}\right) = q = 660.$$

Again, to determine the value of $u\,x\,y\,z$: To the biquadrate of the first equation, add eight times the product of the first and third, and also three times the square of the second; from this subtract six times the square of the first multiplied by the second, plus six times the last, and the result divided by 24, will give

$$u\,x\,y\,z = \left(\frac{a^4 + 8ac + 3b^2 - 6a^2b - 6d}{24}\right) = 700 = z.$$

Now, by the methods above described, we have the four following equations,

$$\begin{aligned}
u + x + y + z - a &= 0\\
ux + uy + uz + xy + xz + yz - p &= 0\\
uxy + uyz + uxz + xyz - q &= 0\\
u\,x\,y\,z - r &= 0
\end{aligned}$$

And, if we multiply the first by u^3, the second by $-u^2$, the third by u, and the fourth by -1, we shall obtain these results,

$$\begin{aligned}
u^4 + u^3x + u^3y + u^3z - au^3 &= 0\\
-u^3x - u^3y - u^3z - u^2xy - u^2xz - u^2yz + u^2p - &= 0\\
+u^2xy + u^2xz + u^2yz + uxyz - uq &= 0\\
-uxyz + r &= 0
\end{aligned}$$

And, by addition, $u^4 - a\,u^3 + p\,u^2 - q\,u + r = 0$. From whence u is found $= 2$. Then, by known methods, x, y, and z, are determined to be respectively equal to 5, 7, and 10.

SOLUTION TO QU. 4. By *Mr. R. Tagart, New-York.*

Prob. 4. Given the sum of four numbers $= 24$, the sum of

their squares = 178, the sum of their cubes = 1476, and the sum of their fourth powers = 13042, to determine the numbers.

Assume $x - r$, $x + r$, $y - s$, and $y + s$, for the numbers; also put $xy = p$. Then, by the nature of the question

$$x + y = 12 = a$$
$$x^2 + y^2 + r^2 + s^2 = 89 = b$$
$$x^3 + 3r^2 x + y^3 + 3s^2 y = 738 = c$$
$$x^4 + 6r^2 x^2 + r^4 + y^4 + 6s^2 y^2 + s^4 = 6521 = d$$

Now, since $x = \frac{a - \sqrt{(a^2 - 4p)}}{2}$ and $y = \frac{a + \sqrt{(a^2 - 4p)}}{2}$

we have by problem 68 of Simpson's Algebra, and the above values of x and y,

$$a^2 - 2p + r^2 + s^2 = b$$
$$a^3 - 3ap + \frac{3as^2 + 3ar^2}{2} + \frac{3s^2 - 3r^2}{2}\sqrt{(a^2 - 4p)} = c$$
$$a^4 - 4a^2 p + 2p^2 + 3a^2 s^2 + 3a^2 r^2 - 6p(s^2 + r^2) + (3a^2 s^2 - 3a^2 r^2)\sqrt{(a^2 - 4p)} + r^4 + s^4 = d.$$

Multiplying the first of these by $\frac{3}{2}a$, subtract the products from the second, and there remains $-\frac{1}{2}a^3 + \frac{3s^2 - 3r^2}{2}\sqrt{(a^2 - 4p)} = c - \frac{3}{2}ab$; from which $3(s^2 - r^2)\sqrt{(a^2 - 4p)} = a^3 - 3ab + 2c = 0$. Consequently, one or both the factors $(s^2 - r^2)$, $\sqrt{(a^2 - 4p)}$ are equal to 0. If *both*, then $s = r$, $x = 6$, $y = 6$, $p = 36$, $r = s = \sqrt{17}$, and the numbers sought would be $6 - \sqrt{17}$, $6 + \sqrt{17}$, and $6 - \sqrt{17}$ $6 + \sqrt{17}$. But these, it will readily be found, do not answer the last condition of the problem; the sum of their 4th powers being 12817.

Again, if we take $\sqrt{(a^2 - 4p)} = 0$, or $p = 36$, then $x = 6$, $y = 6$, and the 4th power equation, omitting $(3a^2 s = 3a^2 r^2)\sqrt{(a^2 - 4p)} = 0$ becomes 216, $(s^2 + r^2) + r^4 + s^4 = d - a^4 + 4a^2 p - 2p^2 = 3929$. From this subtract 12 times the 3d power equation, (omitting the quantity = 0) and there re-

mains $r^4 + s^4 = 257$. Subtract this from $(s^2 + r^2)^2 = (b - a^2 + 2p)^2 = 289$, and we have $2r^2 s^2 = 32$. This, subtracted from $r^4 + s^4 = 257$, gives $s^4 - 2s^2 r^2 + r^4 = 225$; or $s^2 - r^2 = 15$. Also, since $s^2 + r^2 = b - a^2 + 2p = 17$, we obtain $s = 4$, $r = 1$, $x - r = 5$, $x + r = 7$, $y - s = 2$, and $y + s = 10$.

In like manner, if $s^2 - r^2 = 0$, or $s = r$, then the first equation being $x + y = a$, the 2d becoming $x^2 + y^2 = b - 2r^2$, and the 4th, $x^4 + y^4 = d - 6br^2 + 10r^4$: from the first and second of these, $2x^2 y^2 = \frac{(a^2 - b + 2r^2)^2}{2}$; which subtracted from the square of the second, gives $(x^4 + y^4) = (b - 2r^2)^2 - \frac{(a^2 - b + 2r^2)^2}{2} = d - 6br^2 + 10r^4$; this reduced, becomes $r^4 - \frac{17}{2} r^2 = -\frac{225}{16}$: from which $r = \frac{5}{2}$ or $\frac{3}{2}$. Hence $x^2 + y^2 = b - 2r^2 = 76\frac{1}{2}$, or $84\frac{1}{2}$; which subtracted from $(x + y)^2 = a^2$ gives $2xy = 67\frac{1}{2}$, or $59\frac{1}{2}$, and this taken from $x^2 + y^2 = 76\frac{1}{2}$, or $84\frac{1}{2}$, leaves $x^2 - 2xy + y^2 = 9$, or 25. Hence $y - x = 3$, or 5; and since $y + x = 12$, we have $y = 7\frac{1}{2}$, or $8\frac{1}{2}$, and $x = 4\frac{1}{2}$, or $3\frac{1}{2}$. From the last of these values of r, x, and y, we have the numbers in order $x - r = 2$, $x + r = 5$, $y - s = 7$, and $y + s = 10$.

Qu. 4. Answered by *Mr. O'Shannessy, Albany.*

Let half the sum of each pair of numbers $= 6 = a$, and half their difference $= x$, and y, respectively. Then by the question $(a + x)^2 + (a - x)^2 + (a + y)^2 + (a - y)^2 = 178 = 2b$, or $4a^2 + 2x^2 + 2y^2 = 2b$; hence $x^2 + y^2 = b - 2a^2 = 17 = r$, and $y^2 = r - x^2$. Again by the question, $(a + x)^4 + (a - x)^4 + (a + y)^4 + (a - y)^2 = 13042 = 2d$, or $4a^4 + 2x^4 + 12a^2 x^2 + 2y^4 + 12a^2 y^2 = 2d$, and hence $x^4 + 6a^2 x^2 + y^4 + 6a^2 y^2 = d - 2a^4 = 3929 = e$. Now, by substituting the value of y^2, found above, we get $x^4 + 6a^2$

$x^2 + r^2 - 2rx^2 + x^4\ 6a^2 r - 6a^2 x^4 = e$, or $x^4 - r x^2 = \frac{e - 6a^2 r - r^2}{2} = -16 = -p$; whence, by completing the square, &c. we have $x^2 = \frac{1}{2} r - \sqrt{(\frac{1}{4} r^2 - p)} = 1$, also $y^2 = r - x^2 = 16$; therefore, $y = 4$, and the required numbers are 2, 5, 7, and 10. Note. It is well known, that when $a + x$, $a - x$, $a + y$, and $a - y$, denote four numbers, the sum of their squares divided by 2, gives $x^2 + y^2 = \frac{1}{2} b - 2a^2$; and the sum of their cubes divided by $6a$, gives $x^2 + y^2 = (c - 4a^3) \div 6a$; when $4a$ is their sum, b = sum of their square, &c. the sum of their cubes. Therefore, $\frac{1}{2} b - 2a^2 = (c - 4a^3) \div 6a$; finding then, that the numbers 24, 178, and 1476, succeed in this expression, I conclude that the regular numbers must have the same property.

Answers to the Questions proposed in No. 1, to be answered in No. 3.

Qu. 6. Answered by *Mr. P. Nolan.*

Let x denote the number of apples; then $\frac{1}{2}(x - 22)$ = what *A* took the second time, and by the question $5 + \frac{1}{2}(x - 22)$ = *A*'s share = $\frac{1}{2} x$; this equ. reduced, gives $x = 36$.

Qu. 7. Answered by *Y*, and *Mr. Wood.*

Because $l.\ x + l.\ y = 3$, therefore, $l\ (x\ y) = 3$; and as 10 is the radix of the common system, $x\ y = 10^3 = 1000$; hence we have the sum and product of two numbers to find those numbers, which are 10 and 100.

Qu. 8. Answered by *Mr. O'Shannessy, Albany.*

From the second equ. $y^3 = (10 - x^3 - x) \div x$, and $y = \sqrt[3]{(10 - x^3 - x) \div x}$; which values being substituted in the

first, gives $\frac{10 - x^3 - x}{x} + x^3 \sqrt[3]{\frac{10 - x^3 - x}{x}} = 10$; whence, by reduction $x^{14} + x^{12} - 10\,x^{11} + x^9 + 33\,x^7 - 30\,x^6 + 363\,x^5 - 660\,x^4 + 1631\,x^3 - 3630\,x^2 + 3300\,x = 1000$; from whence $x = 1$, and $y = 2$; and diff. lat. : dep. : : 2 : 1. Now, the dist. run is $z\sqrt{5}$ (putting z = dep.) theref. $3z - z\sqrt{5} = 229.1796$ miles, per qu. whence $z = 300$, $= 5^\circ$ of a great circle; theref. diff. lat. $= 10^\circ$ and lat. come to $= 50^\circ$, $28'$. Hence, by inverting the common rule for finding the number of miles in any portion of a parallel of lat. we have cos. lat. : rad. : : $5^\circ : 7^\circ.5'$ 13 diff. long. therefore long. come to $= 67^\circ.1'.87$.

Qu. 9. Answered by *Mr. M. O'Connor.*

The difference between the earth's radius and the sun's distance being so great, the required parallax must necessarily be so small, that its tangent may without any sensible error be taken for its corresponding arc. Therefore 95000000 : 3960 : : 1 = rad. : ,00004168421 = the arc which measures the required parallax. Now 3.14159 : 180° : : ,00004168421 : 8″. 6, the parallax required.

The answers by *Y*, *Mr. P. Nolan*, and *Mr. Wood*, are as follows:

As 95000000 : rad. : : 3960 : sine 8″. 612.

Qu. 10. Answered by the Proposer, *Analyticus.*

Let a, b, c, be the given sides, m, n, p, the cosines of the given angles opposite fo those sides, and let x, yx, zx, be the three slant sides; then by trigonometry we have the three equations,

$$x^2 + x^2 y^2 - m x^2 y = a^2$$
$$x^2 + x^2 z^2 - n x^2 z = b^2$$
$$x^2 y^2 + x^2 z^2 - p x^2 y z = c^2.$$

Divide the last of them by each of the others, and (putting $\frac{c^2}{a^2} = h$, $\frac{c^2}{b^2} = k$) we have

$$\frac{y^2 + z^2 - pyz}{1 + y^2 - my} = h$$

$$\frac{y^2 + z^2 - pyz}{1 + z^2 - nz} = k$$

which are reducible to the form

$$(1 - h)\, y^2 + z^2 - pyz + hmy - h = 0$$

$$(1 - k)\, z^2 + y^2 - pyz + knz - k = 0.$$

Those two equations are of the second degree, in each of the quantities y and z, and, therefore, by the general rules of elimination, the resulting final equation is only of the fourth degree.

When y and z are known, we obtain x from the equation

$$x^2 + x^2\, y^2 - m\, x^2\, y = a^2,$$

and thus the three slant sides of the pyramid are known; and thence we easily obtain the content of the pyramid. *Mr. O. Shannessy*, and *Y*, of New-Haven, solved this question also geometrically.

Mathematical Questions, to be answered in No. 6.

Qu. 24. By *Mr. M. O'Connor, New-York.*

On a given straight line, to describe a segment of a circle, whose area shall be equal to a given area.

Qu. 25. By *Mr. Rt. Maar, New York.*

A gentleman put 2000 pounds into the hands of a banker, in order to receive a weekly allowance; required, what that allowance must be, so that the whole sum, and its compound interest, at the rate of 4 per cent. per annum, may be entirely exhausted in 7 years.

Qu. 26. By *Mr. W. Putsey.*

Find three whole numbers, such, that their sum may be a square, and further, that the sum of any two, minus the third, may also be a square?

Prize Qu. 27. By *Mr. O. Reynolds*, Professor of Mathematics, Baltimore.

Supposing the earth to be a perfect sphere of 7958 miles in diameter; it is required, to find the area of that part of the surface included between the meridian passing through Baltimore and Dublin, and terminating at the North Pole, and the Rhumb line passing through both places: Baltimore being in latitude 39° 23′ N. longitude 76° 50′ W., and Dublin in latitude 53° 22′ N. longitude 6° 17′ W.

The following is a list of the names of gentlemen, who answered the questions proposed in No. 1, and which were to be answered in No. 3.

Analyticus, New-York, . . .	6.	7.	8.	9.	10.
Mr. Rt. Maar, New-York, . . .	6.	7.	8.	9.	10.
Mr. P. Nolan, New-York, . .	6.	7.	8.	9.	
Mr. M. O'Connor, New-York, . .	6.	7.	8.	9.	10.
Mr. Tomlinson, New-York, . .	6.	7.	8.	9.	10.
Tamerlane, ...	6.	7.	8.	9.	10.
Mr. O. Shannessy, Albany, . .	6.	7.	8.	9.	10.
Mr. R. Tagart, New-York, . . .	6.	7.	8.	9.	10.
Mr. Wood, New-York, . . .	6.	7.	8.	9.	
Y., New-Haven, ...	6.	7.	8.	9.	10.
Zero, ...	6.	7.	8.	9.	10.

This last gentleman's letter came too late.